Table of Contents:

Abstract

The following report is a detailed analysis of a particular six-bar system. Included in the report is analysis of position, velocity, and acceleration for each link in the mechanism in addition to the calculation of the radius of curvature, the velocity and acceleration of the center of curvature of point P. Also, a scale drawing that includes the instant centers of the mechanism and analysis based off of it is included. Solving for these variables using the Newton Raphson method, the Instant Center method, and the Finite Difference method demonstrates how similar values can be calculated by focusing on different portions of the known information.

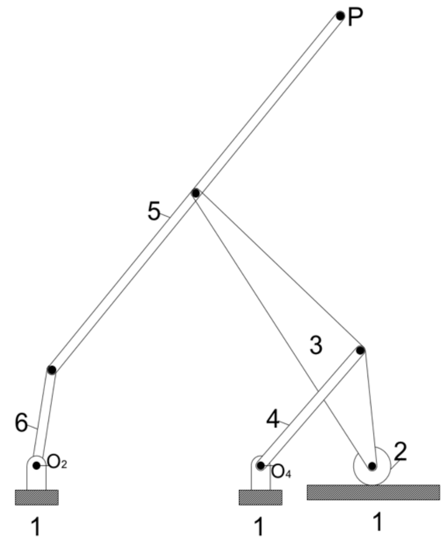

Figure 1: The mechanism in its original position

Introduction

In this mechanism a wheel is rolling, without slip, away from a joint O_4, and the wheel center rolls from 75 mm from O_4 to 150 mm from O_4. The process of doing analysis of this six-bar system consists of creating Vector Loops, performing Position Analysis, and then using the first and second order kinematic coefficients to calculate angular velocity and angular acceleration. Point P, the farthest point on link 5 from link 6 (See Figure 1 for visual), has a radius of curvature, center of curvature, Unit Normal vector, and Unit Tangent vector that are also calculated. The following report will walk through each of these steps, including all equations used to make these calculations, and all results achieved.

Six-Bar Analysis

Vector Loops:

To complete position analysis of a system, Vector Loops and Vector Loop Equations must be created. Vectors in the loops are generally in the direction of a particular link and at the length of that same link. For every vector loop there can only be two unknowns, or if there are more than two unknowns, there must be a constraint to bring the number of unknowns down to two. The first vector loop used for analysis is shown in Figure 2. Following the loop in a counter-clockwise direction creates the basic vector loop equation:

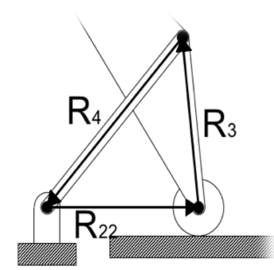

Figure 2: Vector Loop #1

$$R_4 + R_3 + R_{22} = 0 \text{ (Eqn. 1.)}$$

This basic equation can be expanded to read:

$$R_4\cos(\theta_4) + R_3\cos(\theta_3) + R_{22}\cos(\theta_{22}) = 0 \text{ (Eqn. 1a.)}$$
$$R_4\sin(\theta_4) + R_3\sin(\theta_3) + R_{22}\sin(\theta_{22}) = 0 \text{ (Eqn. 1b.)}$$

Using a similar process, the second vector loop, shown in Figure 3., can be used to determine the second vector loop equation. Following this loop provides the equation:

$$R_{33} + R_5 + R_6 + R_{22} + R_1 = 0 \text{ (Eqn. 2.).}$$

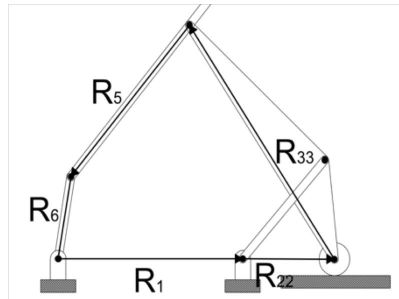

Figure 3: Vector Loop Equation #2

This equation can be expanded to become:

$$R_{33}\cos(\theta_{33}) + R_5\cos(\theta_5) + R_6\cos(\theta_6) + R_{22}\cos(\theta_{22}) + R_1\cos(\theta_1) = 0 \text{ (Eqn. 2a.)}$$
$$R_{33}\sin(\theta_{33}) + R_5\sin(\theta_5) + R_6\sin(\theta_6) + R_{22}\sin(\theta_{22}) + R_1\sin(\theta_1) = 0 \text{ (Eqn. 2b.)}$$

Position Analysis:
Because the problem statement provides the length of R_4, R_3, and R_{22}, and θ_{22} will always be parallel to the ground ($\theta_{22} = 0°$), the only unknowns in Eqn. 1a. and Eqn. 1b. are θ_3 and θ_4. Using the Newton-Raphson method, where guesses will be updated until values are within a tolerance of 0.01°, θ_3 and θ_4 are calculated. This method is used for all R_{22} values from 75mm to 150 mm, and now all values in the first vector loop equation are known. In Eqn. 2a. and Eqn. 2b., R_{33}, $R_5, R_6, R_1, R_{22}, \theta_{22}$,and θ_1 are all known ($\theta_1 = 0°$), and $\theta_{33} = \theta_3$ + a constant because both θ_{33} and θ_3 end on the same link, therefore having a constant angle value between them. This being the case, the only unknowns are θ_5 and θ_6. Solving numerically, θ_5 and θ_6 are calculated and all information in Eqn. 2a. and Eqn. 2b. is now known. The tabulated data for position analysis as R_2 moves from 75 mm to 150 mm from O_4 can be found in Appendix A.1 and the plot of all link positions based off of the input link length is shown in Figure 4. As this plot shows, when the length of R_{22} increases, the values of θ_3 and θ_6 increase, while the values of θ_4 and θ_5 decrease. None of these plots are truly linear, but θ_3 for $75mm \leq R_{22} \leq 150mm$ appears to be far more linear than the rest of them.

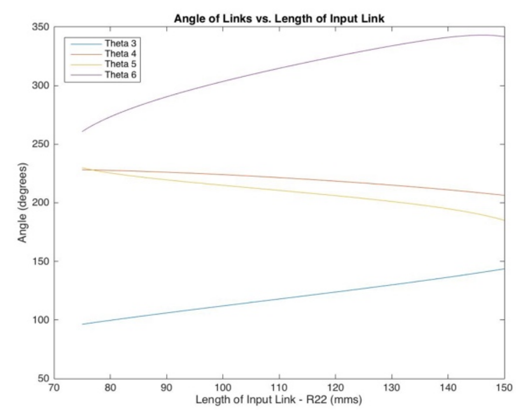

Figure 4: Angle of Links vs. Length of Input Link

Number of Rotations:
To determine the number of rotations the wheel will make when $75mm \leq R_{22} \leq 150mm$ the rolling contact equation must be used.

$$\Delta R_{22} = \rho_2\theta_2\ (Eqn.3). ==> \ \theta_{22} = \frac{\Delta R_2}{\rho_2} = \frac{75\ mm}{12.5\ mm} = 6\frac{mm}{mm} = 343.77°\ or\ 0.955\ rotations$$

Though the wheel will not complete an entire rotation over the extent of point A's displacement it will get quite close to doing so.

Kinematic Coefficients:

To determine the first-order kinematic coefficients for link 3 and 4, the derivatives of Eqn. 1a. and Eqn. 1b. are taken and the values in each equation are separated into groups of the two unknowns and then a third group of values that are able to be calculated (See Eqn. 4a. and Eqn. 4b. for an example).

$$-R_4\sin(\theta_4)\,\theta_4' - R_3\sin(\theta_3)\theta_3' = -\cos(\theta_{22}) \text{ (Eqn. 4a.)}$$
$$R_4\cos(\theta_4)\theta_4' + R_3\cos(\theta_3)\theta_3' = -\sin(\theta_{22}) \text{ (Eqn. 4b.)}$$

$$-R_5\sin(\theta_5)\theta_5' - R_6\sin(\theta_6)\theta_6' = -\cos(\theta_{22}) + R_{33}\sin(\theta_{33})\theta_3' \text{ (Eqn. 4c.)}$$
$$R_5\cos(\theta_5)\theta_5' + R_6\cos(\theta_6)\theta_6' = -\sin(\theta_{22}) - R_{33}\cos(\theta_{33})\theta_3' \text{ (Eqn. 4d.)}$$

Cramer's Rule is applied to the rearranged equations to determine the values of θ_3' and θ_4'. A similar calculation is completed for θ_5' and θ_6' using Eqn. 2a. and 2b. (See Eqn. 4c. and Eqn. 4d.), and the results for θ_3', θ_4', θ_5', and θ_6' over the course of $75mm \leq R_{22} \leq 150mm$ can be found in Appendix A.2. Figure 5 shows how θ changes in all links over the course of the machines movement. As R_{22} increases, $\Delta\theta_3$ remains positive and increases slightly in magnitude over the extent of the mechanism's movement, $\Delta\theta_4$ increases in magnitude negatively, $\Delta\theta_5$ remains negative but is parabolic, and $\Delta\theta_6$ decreases and moves from a positive value to a negative one.

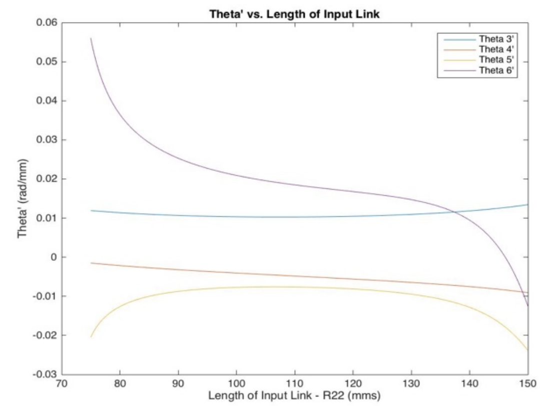

Figure 5: Theta' vs. Length of Input Link

$$-R_4\sin(\theta_4)\,\theta_4'' - R_3\sin(\theta_3)\theta_3'' = R_4\cos(\theta_4){\theta_4'}^2 + R_3\cos(\theta_3){\theta_3'}^2 \text{ (Eqn. 5a.)}$$
$$R_4\cos(\theta_4)\theta_4'' + R_3\cos(\theta_3)\theta_3'' = R_4\sin(\theta_4){\theta_4'}^2 + R_3\sin(\theta_3){\theta_3'}^2 \text{ (Eqn. 5b.)}$$

$$-R_5\sin(\theta_5)\theta_5'' - R_6\sin(\theta_6)\theta_6'' = R_5\cos(\theta_5){\theta_5'}^2 + R_6\cos(\theta_6){\theta_6'}^2 + R_{33}\cos(\theta_{33}){\theta_3'}^2 + R_{33}\sin(\theta_{33})\theta_3'' \text{ (Eqn. 5c.)}$$
$$R_5\cos(\theta_5)\theta_5'' + R_6\cos(\theta_6)\theta_6'' = R_5\sin(\theta_5){\theta_5'}^2 + R_6\sin(\theta_6){\theta_6'}^2 + R_{33}\sin(\theta_{33}){\theta_3'}^2 - R_{33}\cos(\theta_{33})\theta_3'' \text{ (Eqn. 5d.)}$$

A similar process is used to find second-order kinematic coefficients. The derivative of the derivative equations are taken (See Eqn. 5a., Eqn. 5b., Eqn. 5c., and Eqn. 5d.) and Cramer's Rule is applied to determine the values of θ_3'', θ_4'', θ_5'' and θ_6''. The tabulated data of all θ_n'' can be found in Appendix A.3, and this data can be seen plotted in Figure 6. θ_3'' and θ_4'' begin linear and then break off after R_{22} crosses the middle point of the move (when R_{22}= 112.5 mms), θ_3'' becoming positive and θ_4'' remaining negative. θ_5'' on the other hand appears to be a cubic function, and θ_6'' appears to be a wide parabola.

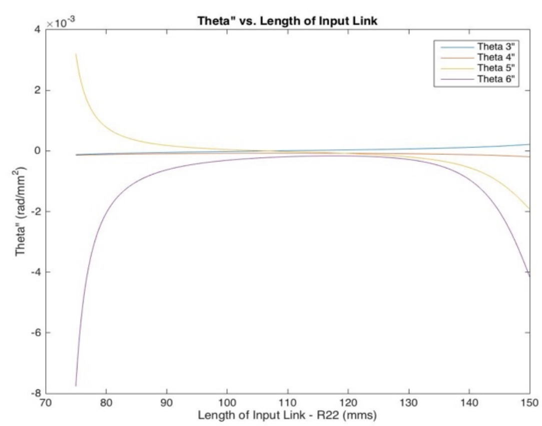

Figure 6: Theta" vs. Length of Input Link

Angular Velocity Results:
The angular velocity of all links other than link 2 can be calculated:
when $R_{22} < 112.5$ mm by using:

$$\omega_n = \theta_n' * \sqrt{2\ddot{R}(R_{22} - 0.075)}\ ,\ \ddot{R} = 0.125\ \frac{m}{s^2} \text{ (Eqn. 6a.)}$$

or when $R_{22} \geq 112.5$ mm by using:

$$\omega_n = \theta_n' * \sqrt{2\ddot{R}(R_{22} - 0.15)}\ ,\ \ddot{R} = -0.125\ \frac{m}{s^2} \text{ (Eqn. 6b.)}$$

The angular velocity of link 2 can be calculated:
when $R_{22} < 112.5$ mm by using:

$$\omega_2 = -\sqrt{2\ddot{R}(R_{22} - 0.075)}\ ,\ \ddot{R} = 0.125\ \frac{m}{s^2} \text{ (Eqn. 6c.)}$$

or when $R_{22} \geq 112.5$ mm by using:

$$\omega_2 = -\sqrt{2\ddot{R}(R_{22} - 0.15)}\ ,\ \ddot{R} = -0.125\ \frac{m}{s^2} \text{ (Eqn. 6d.)}$$

The results of each link's angular velocity can be found in Appendix A.3, and the plot of each link's angular velocity against the input length is displayed in Figure 7. Each of the angular velocity curves have a similar overall shape, but ω_3 and ω_6 are positive (they're moving counterclockwise), and ω_3 and ω_4 are negative (they're moving clockwise). ω_6 has a larger magnitude angular velocity than ω_3 for the majority of the move, and ω_5 has a larger magnitude angular velocity than ω_4.

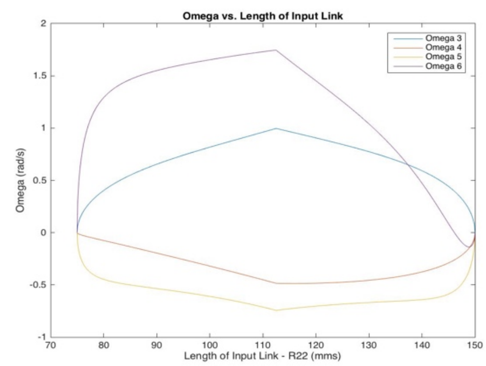

Figure 7: Omega vs. Length of Input Link

Angular Acceleration Results:
The angular accelerations of links 3, 4, 5 and 6 are calculated using the standard form equation:

$$\alpha_n = \theta_n'' * \dot{R}^2 + \theta_n' * \ddot{R} \text{ (Eqn. 7)}$$

which for R < 112.5 mm is:

$$\alpha_n = 2\theta_n'' * \ddot{R}(R_{22} - 0.075) + \theta_n' * \ddot{R},\ \ddot{R} = 0.125\ \frac{m}{s^2} \text{ (Eqn. 7a.)}$$

and for R ≥ 112.5 mm is:

$$\alpha_n = 2\theta_n'' * \ddot{R}(R_{22} - 0.15) + \theta_n' * \ddot{R},\ \ddot{R} = -0.125\ \frac{m}{s^2} \text{ (Eqn. 7b.)}$$

The angular acceleration of link 2 can be determined by the following this equation where ρ is the radius of the wheel.
when R < 112.5 mm:

$$\alpha_2 = \frac{-\ddot{R}}{\rho},\ \ddot{R} = 0.125\ \frac{m}{s^2} \text{ (Eqn. 7c.)}$$

when R ≥ 112.5 mm is:

$$\alpha_2 = \frac{-\ddot{R}}{\rho},\ \ddot{R} = -0.125\ \frac{m}{s^2} \text{ (Eqn. 7d.)}$$

All data for the angular accelerations of the links can be found in Appendix A.5., and the plot of the angular accelerations in relation to the input length can be viewed in Figure 8. As can be seen in Figure 8, the links are almost mirrored about the midpoint of the move (R_{22}= 112.5 mms). α_3 and α_4 have a pretty steady low sloped angular acceleration change disregarding the discrepancy near 112.5 mm, and the curves of α_5 and α_6 follow the general path of the first two curves with less strict edges.

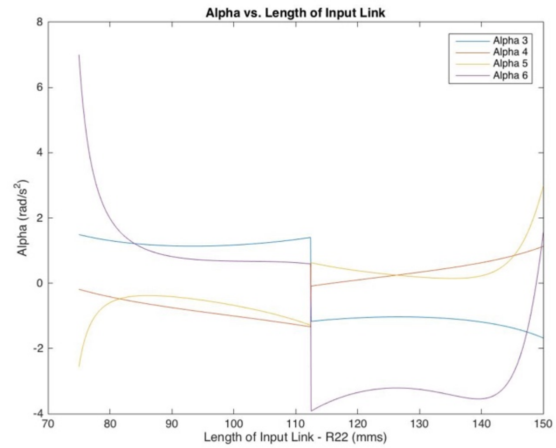

Figure 8: Alpha vs. Length of Input Link

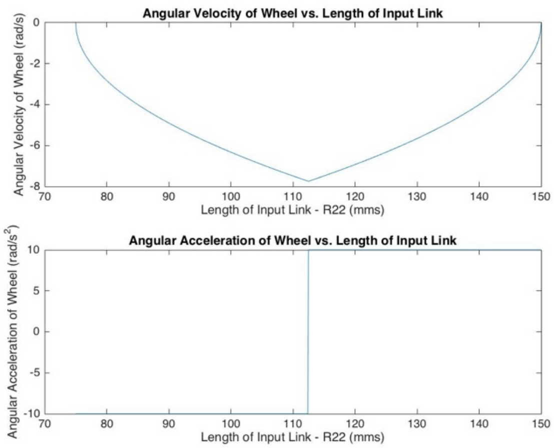

Figure 9: Angular Velocity and Angular Acceleration vs. Length of Input Link

In Figure 9 it is evident that link 2 is rotating clockwise (all ω_2 values are negative) and that the acceleration is either -10 $\frac{rad}{s^2}$ or 10 $\frac{rad}{s^2}$ for the majority of the move.

Finite Distance Method:

To ensure that the values for θ' and θ'' by the Matlab code are correct, the finite distance method can be used to check percentage errors of the code. Below are finite distance checks for the input lengths of R_{22}= 107.5 mm and R_{22}= 130 mm of the first and second order kinematic coefficients. The base equation that will be followed is:

$$\theta_n' = \frac{\Delta\theta_n}{\Delta InputLength} \text{ (Eqn. 8)}$$

Values needed to complete all calculations:

From Appendix A.1.:

$R_{22}= 105$ $\theta_3 = 114.97$ $\theta_4= 222.83$ $\theta_5= 212.76$ $\theta_6= 309.32$

$R_{22}= 110$ $\theta_3 = 117.92$ $\theta_4= 221.51$ $\theta_5= 210.59$ $\theta_6= 314.76$

$R_{22}= 127.5$ $\theta_3 = 128.41$ $\theta_4= 215.99$ $\theta_5= 202.46$ $\theta_6= 331.77$

$R_{22}= 132.5$ $\theta_3 = 131.54$ $\theta_4= 214.15$ $\theta_5= 199.74$ $\theta_6= 335.98$

From Appendix A.2.:

$R_{22}= 105$ $\theta_3' = 0.0102727$ $\theta_4'= -0.004436$ $\theta_5'= -0.007579$ $\theta_6'= 0.019560$

$R_{22}= 110$ $\theta_3' = 0.0102725$ $\theta_4'= -0.004817$ $\theta_5'=-0.007609$ $\theta_6'= 0.018497$

$R_{22}= 127.5$ $\theta_3' = 0.010797$ $\theta_4'= -0.006218$ $\theta_5'= -0.009007$ $\theta_6'= 0.015352$

$R_{22}= 132.5$ $\theta_3' = 0.011127$ $\theta_4'= -0.006688$ $\theta_5'= -0.010010$ $\theta_6'= 0.013916$

From Appendix A.3.:
R_{22}= 130 $\theta_3'' = 0.0000658$ θ_4''= -0.0000938 θ_5''= -0.0001986 θ_6''= 0.0002827
$\Delta InputLength = \Delta R_{22} = 5mm$

<u>Check for R_{22}= 107.5 mm</u>
First-Order Kinematic Coefficient Check:
First, the change in angle must be calculated.

$$\Delta\theta_3 = \theta_3(110) - \theta_3(105) = 117.92 - 114.97 = 2.95 \text{ degrees}$$
$$\Delta\theta_4 = \theta_4(110) - \theta_4(105) = 221.51 - 222.83 = -2.17 \text{ degrees}$$
$$\Delta\theta_5 = \theta_5(110) - \theta_5(105) = 210.59 - 212.76 = -1.09 \text{ degrees}$$
$$\Delta\theta_6 = \theta_6(110) - \theta_6(105) = 314.76 - 309.32 = 5.44 \text{ degrees}$$

The change in angle is then turned into radians and divided by the change in input length.

$$\theta_3' = \frac{\Delta\theta_3 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{0.05148\, rad}{5\, mm} = 0.010297 \frac{rad}{mm}$$
$$\theta_4' = \frac{\Delta\theta_4 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{-0.02304\, rad}{5\, mm} = -0.004671 \frac{rad}{mm}$$
$$\theta_5' = \frac{\Delta\theta_5 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{-0.03787\, rad}{5\, mm} = -0.007575 \frac{rad}{mm}$$
$$\theta_6' = \frac{\Delta\theta_6 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{0.09495\, rad}{5\, mm} = 0.018989 \frac{rad}{mm}$$

Finally, each calculated θ' value is compared to the θ' value given to Appendix A.2. to determine the percentage error of each calculation.

$\theta_3'(numerical) = 0.0103 \frac{rad}{mm}$ Error = 0.025%
$\theta_4'(numerical) = -0.0046 \frac{rad}{mm}$ Error = 0.166%
$\theta_5'(numerical) = -0.0076 \frac{rad}{mm}$ Error = 0.333%
$\theta_6'(numerical) = 0.0190 \frac{rad}{mm}$ Error = 0.057%

As all percentage errors are quite small, a similar process is followed using the θ' values from the Matlab code can be deemed to be running correctly.

Second-Order Kinematic Coefficient Check:

$$\Delta\theta_3' = \theta_3'(110) - \theta_3'(105) = 0.0102725 - 0.0102727 = -2*10\text{^}-7 \text{ rad/mm}$$
$$\Delta\theta_4' = \theta_4'(110) - \theta_4'(105) = -0.004817 - (-0.004436) = -3.81*10\text{^}-4 \text{ rad/mm}$$
$$\Delta\theta_5' = \theta_5'(110) - \theta_5'(105) = -0.007609 - (-0.007579) = -3*10\text{^}-5 \text{ rad/mm}$$
$$\Delta\theta_6' = \theta_6'(110) - \theta_6'(105) = 0.018497 - 0.019560 = -0.001063 \text{ rad/mm}$$

$$\theta_3'' = \frac{\Delta\theta_3'}{\Delta R_{22}} = \frac{-2 * 10^{-7} rad/mm}{5\, mm} = -4 * 10\text{^} - 8 \frac{rad}{mm^2}$$
$$\theta''_4 = \frac{\Delta\theta_4'}{\Delta R_{22}} = \frac{-3.81*10\text{^}-4\ \ rad/mm}{5\, mm} = -0.004671 \frac{rad}{mm}$$
$$\theta_5'' = \frac{\Delta\theta_5'}{\Delta R_{22}} = \frac{-3 * 10\text{^} - 5\, rad/mm}{5\, mm} = -6 * 10\text{^} - 6 \frac{rad}{mm}$$
$$\theta_6'' = \frac{\Delta\theta_6'}{\Delta R_{22}} = \frac{-0.001063 \text{ rad/mm}}{5\, mm} = -2.126 * 10\text{^} - 4 \frac{rad}{mm}$$

$$\theta''_3(numerical) = -0.000000036\ \frac{rad}{mm} \qquad \text{Error} = 10.00\%$$
$$\theta_4''(numerical) = -0.000076067\ \frac{rad}{mm} \qquad \text{Error} = 0.175\%$$
$$\theta_5''(numerical) = -0.000006113\ \frac{rad}{mm} \qquad \text{Error} = 1.849\%$$
$$\theta_6''(numerical) = -0.000211736\ \frac{rad}{mm} \qquad \text{Error} = 0.408\%$$

Once again, all error values are quite small. The percentage error for θ''_3 is allowable because the value itself is so small that it would be very difficult to be exact.

Check for R_{22}= 130 mm

First-Order Kinematic Coefficient Check:

$$\Delta\theta_3 = \theta_3(132.5) - \theta_3(127.5) = 131.54 - 128.41 = 3.13 \text{ degrees}$$
$$\Delta\theta_4 = \theta_4(132.5) - \theta_4(127.5) = 214.15 - 215.99 = -1.84 \text{ degrees}$$
$$\Delta\theta_5 = \theta_5(132.5) - \theta_5(127.5) = 199.74 - 202.46 = -2.72 \text{ degrees}$$
$$\Delta\theta_6 = \theta_6(132.5) - \theta_6(127.5) = 335.98 - 331.77 = 4.21 \text{ degrees}$$

$$\theta_3' = \frac{\Delta\theta_3 * \left(\frac{\pi}{180}\right)}{\Delta R_{22}} = \frac{0.05463\ rad}{5\ mm} = 0.010926\ \frac{rad}{mm}$$
$$\theta_4' = \frac{\Delta\theta_4 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{-0.032114\ rad}{5\ mm} = -0.006423\ \frac{rad}{mm}$$
$$\theta_5' = \frac{\Delta\theta_5 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{-0.047473\ rad}{5\ mm} = -0.009495\ \frac{rad}{mm}$$
$$\theta_6' = \frac{\Delta\theta_6 * (\frac{\pi}{180})}{\Delta R_{22}} = \frac{0.073478\ rad}{5\ mm} = 0.014696\ \frac{rad}{mm}$$

$$\theta_3'(numerical) = 0.0109\ \frac{rad}{mm} \qquad \text{Error} = 0.236\%$$
$$\theta_4'(numerical) = -0.0064\ \frac{rad}{mm} \qquad \text{Error} = 0.355\%$$
$$\theta_5'(numerical) = -0.0095\ \frac{rad}{mm} \qquad \text{Error} = 0.057\%$$
$$\theta_6'(numerical) = 0.0147\ \frac{rad}{mm} \qquad \text{Error} = 0.029\%$$

Second-Order Kinematic Coefficient Check:

$$\Delta\theta_3' = \theta_3'(132.5) - \theta_3'(127.5) = 0.011127 - 0.010797 = 3.3*10^{-4} \text{ rad/mm}$$
$$\Delta\theta_4' = \theta_4'(132.5) - \theta_4'(127.5) = -0.006688 - (-0.006218) = -4.7*10^{-4} \text{ rad/mm}$$
$$\Delta\theta_5' = \theta_5'(132.5) - \theta_5'(127.5) = -0.010010 - (-0.009007) = -0.001003 \text{ rad/mm}$$
$$\Delta\theta_6' = \theta_6'(132.5) - \theta_6'(127.5) = 0.013916 - 0.015352 = -0.001436 \text{ rad/mm}$$

$$\theta_3'' = \frac{\Delta\theta_3'}{\Delta R_{22}} = \frac{3.3 * 10\text{^} - 4\ \text{rad/mm}}{5\ mm} = 6.6\text{^} - 5\ \frac{rad}{mm^2}$$

$$\theta''_4 = \frac{\Delta\theta_4'}{\Delta R_{22}} = \frac{-4.7*10\text{^}-4\ \text{rad/mm}}{5\ mm} = \text{-9.4*10^-5}\ \frac{rad}{mm}$$

$$\theta_5'' = \frac{\Delta\theta_5'}{\Delta R_{22}} = \frac{-0.001003\ \text{rad/mm}}{5\ mm} = -2.006 * 10\text{^} - 4\ \frac{rad}{mm}$$

$$\theta_6'' = \frac{\Delta\theta_6'}{\Delta R_{22}} = \frac{-0.001436\ \text{rad/mm}}{5\ mm} = -2.872 * 10\text{^} - 4\ \frac{rad}{mm}$$

$$\theta''_3(numerical) = 0.0000658\ \frac{rad}{mm} \quad \text{Error} = 0.304\%$$

$$\theta_4''(numerical) = -0.0000938\ \frac{rad}{mm} \quad \text{Error} = 0.213\%$$

$$\theta_5''(numerical) = -0.0001986\ \frac{rad}{mm} \quad \text{Error} = 1.007\%$$

$$\theta_6''(numerical) = -0.0002827\ \frac{rad}{mm} \quad \text{Error} = 1.592\%$$

Because all percentage error values are relatively small, the Newton-Raphson results agree well with the finite difference values, and therefore the values are being calculated correctly.

Point P Analysis

Position Analysis:

The point P is at the end of link 5 is attached to ground through link 6. Therefore, the position vector of point P can be written as:

$$R_P + R_5 + R_6 = 0 \text{ (Eqn. 9.)}$$

where R_P can be separated into its X and Y components:

$X_p = -2R_5 \cos(\theta_5) - R_6 \cos(\theta_6)$ (Eqn. 9a.)

$Y_p = -2R_5 \sin(\theta_5) - R_6 \sin(\theta_6)$ (Eqn. 9b.)

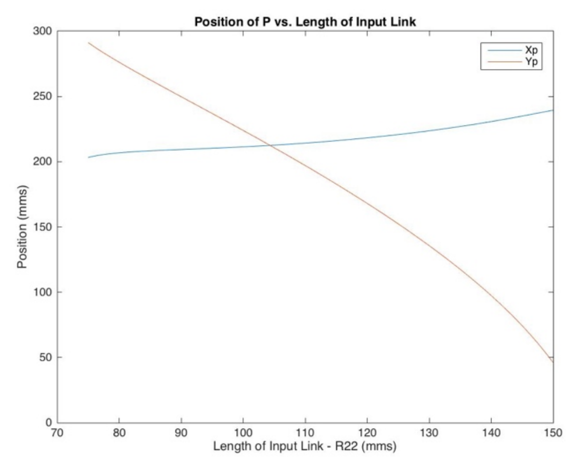

Figure 10: Position of P vs. Length of Input Link

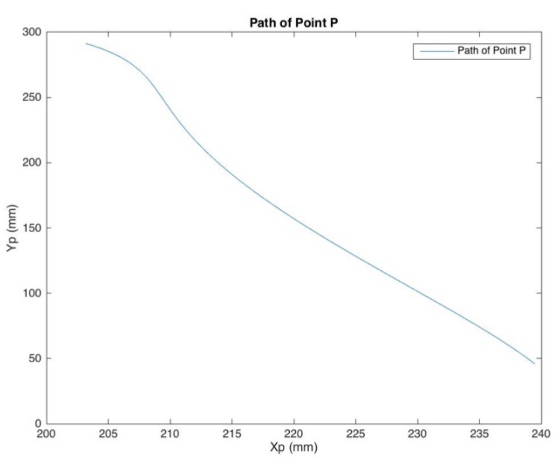

Figure 11: Path of Point P

The value of R_5 is multiplied by 2 because the length from O_2 to point P is twice the distance being called R_5. The position data for point P can be found in Appendix A.6.. Figure 10 shows the relationship of value between X_p and Y_p while Figure 11 shows the path of point P. X_p and Y_p have an inverse relationship which makes point P's path appear almost linear.

Minimum and Maximum Position Values:
When R_{22} = 75 mm, X_p is at a minimum displacement of 203.17 mm, and Y_p is at a maximum displacement of 291.29 mm. In addition, when R_{22} = 150 mm, X_p is at a maximum displacement of 239.45 mm, and Y_p is at a minimum displacement of 45.97 mm (All values from Appendix A.6.)

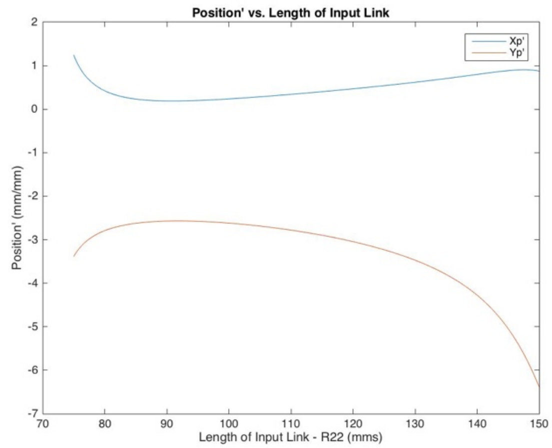

Figure 12: Position' vs. Length of Input Link

Kinematic Coefficients:
The First-Order Kinematic Coefficients for point P's movement can be calculated by taking the derivative of Eqn. 9a. and Eqn. 9b which gives the equations:

$X_p' = 2R_5 \sin(\theta_5)\theta_5' + R_6 \sin(\theta_6)\, \theta_6'$ (Eqn. 10a.)
$Y_p' = -2R_5 \cos(\theta_5)\theta_5' - R_6 \cos(\theta_6)\, \theta_6'$ (Eqn. 10b.)

As seen in Figure 12., X_p' has slight cubic shape while Y_p' is parabolic. R' in the X direction is also a very small number in comparison to R' in the Y direction where the First-Order Kinematic Coefficients range from a minimum of -2.5 mm/mm to a maximum of -7 mm/mm during the move. The tabulated results for the First-Order Kinematic Coefficients of P can be found in Appendix A.6..

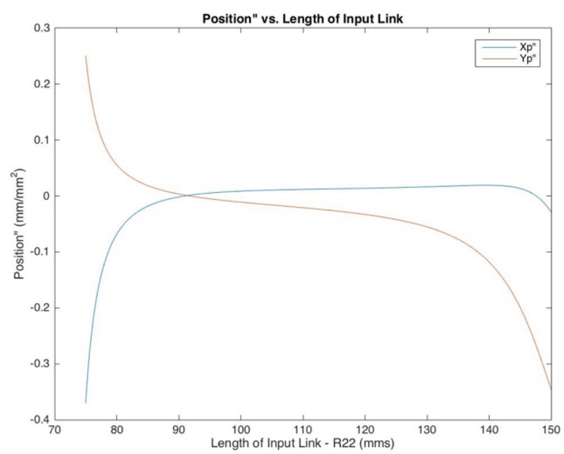

Figure13: Position" vs. Length of Input Link

Second-Order Kinematic Coefficients for point P's movement can be calculated by taking the derivative of Eqn. 10a. and Eqn. 10b which gives the equations:

$X_p'' = 2R_5 \cos(\theta_5)\theta_5'^2 + 2R_5 \sin(\theta_5)\theta_5'' + R_6 \cos(\theta_6)\, \theta_6'^2 + R_6 \sin(\theta_6)\theta_6''$ (Eqn. 11a.)
$Y_p'' = 2R_5 \sin(\theta_5)\theta_5'^2 - 2R_5 \cos(\theta_5)\theta_5'' + R_6 \sin(\theta_6)\, \theta_6'^2 - R_6 \cos(\theta_6)\theta_6''$ (Eqn. 11b.)

The results of these values can also be found in Appendix A.6., and the plot of these values based on the input length and R_{22}''s value shown in Figure 13. The figure shoes that the Second-Order Kinematic Coefficient of point P's position in X has a large slope at the beginning and end of the move, but in the middle of the move the Second-Order Kinematic Coefficient in the X direction is almost constant near 0. In comparison, the Second-Order Kinematic Coefficient of point P's position in the Y direction begins positively with a steep downward slope, crosses through 0 with a low slope, and then increases in slope magnitude once it is negative.

Unit Tangent and Unit Normal Vectors:
To find the Unit Tangent and Unit Normal vectors of point P at all times, it is necessary to first determine R_p' using the standard formula:

$$R_p' = \sqrt{X_p' + Y_p'} \text{ (Eqn. 12)}$$

After R_p' is calculated, Unit Tangent and Unit Normal Vectors in their X and Y components can be calculated using these equations:

$Ut_x = \frac{X_p'}{R_p'}$ (Eqn. 13a.) $\quad$ $Un_x = \frac{-Y_p'}{R_p'}$ (Eqn. 13c.)

$Ut_y = \frac{Y_p'}{R_p'}$ (Eqn. 13b.) $\quad$ $Un_y = \frac{X_p'}{R_p'}$ (Eqn. 13d.)

The tabulated data for the Unit Tangent and Unit Normal vectors of point P can be found in Appendix A.7. As shown by the plot in Figure 14 and Figure 15, Eqn. 13a. and Eqn. 13d. are the same exact calculation, while Eqn. 13b. gives a value very close to -1and Eqn. 13c. gives a value very close to +1.

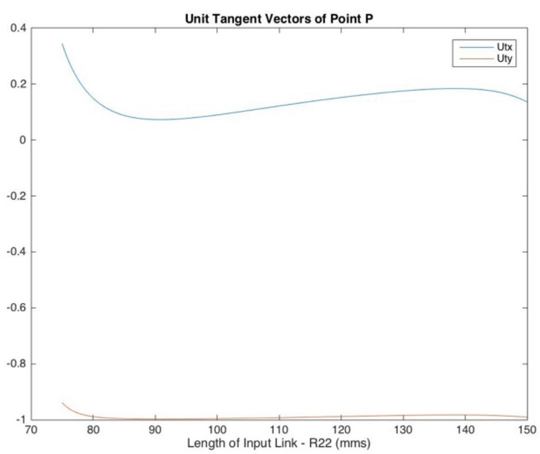

Figure 14: Unit Tangent Vectors of Point P

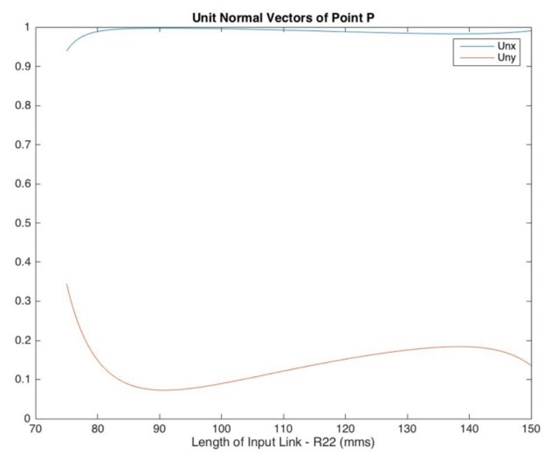

Figure 15: Unit Normal Vectors of Point P

Radius of Curvature and Center of Curvature:

The Radius of Curvature of point P can be determined by using the standard Radius of Curvature formula:

$$\rho_c = \frac{{R_p'}^3}{X_p' * Y_p'' - X_p'' * Y_p'} \text{ (Eqn. 14)}$$

The values for the Radius of Curvature of point P as the length of R_{22} changes can be found in Appendix A.8. Below, Figure 16 shows how the radius of curvature changes. There is a dramatic shift in radius of curvature just after R_{22} becomes larger than 90 mm, and there is another dramatic shift when R_{22} is about 138 mm in length. These shifts are most likely due to a rocking motion in one of the links that would cause the radius of curvature to flip signs.

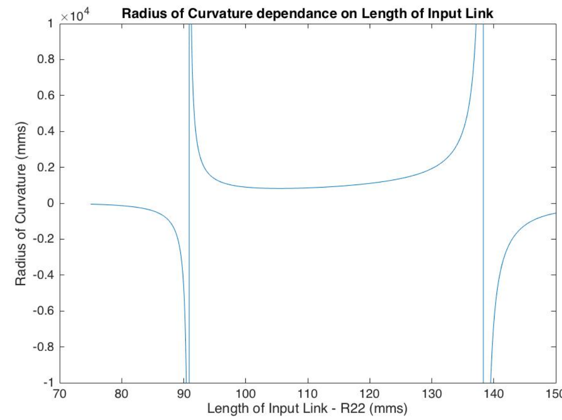

Figure 16: Radius of Curvature dependence on Length of Input Link

The Center of Curvature of point P is found by using the position analysis of point P, the Unit Normal Vector, and Radius of Curvature. The following equation describes the relationship.

$$X_{cc} = X_p + \rho_c * Un_x \text{ (Eqn. 15a.)}$$
$$Y_{cc} = Y_p + \rho_c * Un_y \text{ (Eqn. 15b.)}$$

All recorded values for the center of curvature for $75mm \leq R_{22} \leq 150mm$ can be found in Appendix A.8. Figure 17 describes the dependence of the on center of curvature on R_{22}''s length and Figure 18 shows the relationship between the X and Y components of the center of curvature. Similar to the radius of curvature, there are discrepancies in the path when R_{22} is either at approximately 90 mm or 138 mm.

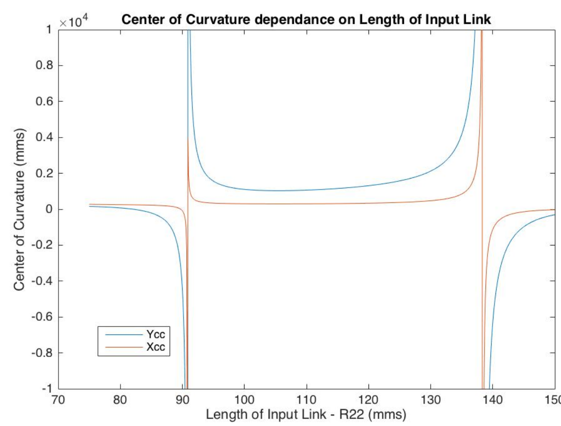

Figure 17: Center of Curvature dependence on Length of Input Link

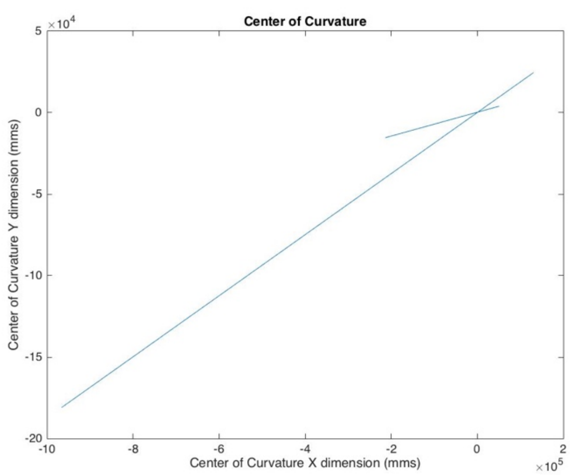

Figure 18: Center of Curvature

Velocity Analysis:
The velocity of point P can be determined by the base formula:

$$V_p = R_p' * \dot{R} \text{ (Eqn. 16)}$$

which when split into its components can be written more specifically as:

(when R_{22} < 112.5 mm)

$$V_{px} = X_p' * \sqrt{2\ddot{R} * (R_{22} - 0.075)} \text{ (Eqn. 16a.) where } \ddot{R} = 0.125 \frac{m}{s^2}$$

$$V_{py} = Y_p' * \sqrt{2\ddot{R} * (R_{22} - 0.075)} \text{ (Eqn. 16b.) where } \ddot{R} = 0.125 \frac{m}{s^2}$$

(when $R_{22} \geq$ 112.5 mm)

$$V_{px} = X_p' * \sqrt{2\ddot{R} * (R_{22} - 0.15)} \text{ (Eqn. 16c.) where } \ddot{R} = -0.125 \frac{m}{s^2}$$

$$V_{py} = Y_p' * \sqrt{2\ddot{R} * (R_{22} - 0.15)} \text{ (Eqn. 16d.) where } \ddot{R} = -0.125 \frac{m}{s^2}$$

The velocity data of point P as R_{22} moves from 75 mm to 150 mm in length can be found in Appendix A.9. Figure 19 shows that the system begins at 0 mm/s for both X and Y, and over the course of the movement, the velocity in the X direction is positive while the velocity in the Y direction is negative. Both values come back to zero at the end of the move, and the magnitude of every Vy value larger than the magnitude of its corresponding Vx value.

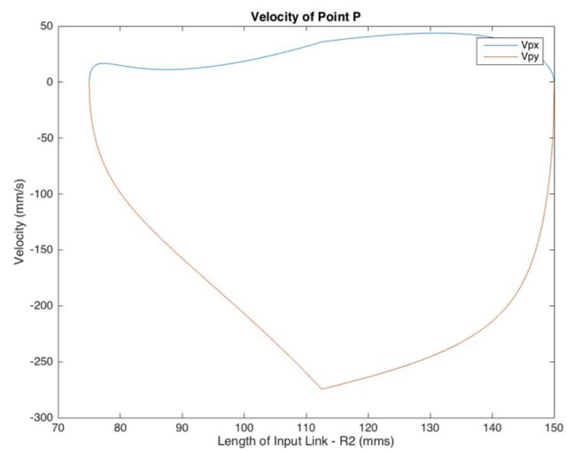

Figure 19: Velocity of Point P

Acceleration Analysis:
The acceleration of point P is determined by the base formula:

$$A_p = R_p' * \ddot{R} + R_p'' * \dot{R} \text{ (Eqn. 17)}$$

which when split into its components can be written more specifically as:

(when R_{22} < 112.5 mm)

$$A_{px} = X_p' * \ddot{R} + X_p'' * \sqrt{2\ddot{R} * (R_{22} - 0.075)} \text{ (Eqn. 17a.) where } \ddot{R} = 0.125 \frac{m}{s^2}$$

$$A_{py} = Y_p' * \ddot{R} + Y_p'' * \sqrt{2\ddot{R} * (R_{22} - 0.075)} \text{ (Eqn. 17b.) where } \ddot{R} = 0.125 \frac{m}{s^2}$$

(when $R_{22} \geq$ 112.5 mm)

$$A_{px} = X_p' * \ddot{R} + X_p'' * \sqrt{2\ddot{R} * (R_{22} - 0.15)} \text{ (Eqn. 17c.) where } \ddot{R} = -0.125 \frac{m}{s^2}$$

$$A_{py} = Y_p' * \ddot{R} + Y_p'' * \sqrt{2\ddot{R} * (R_{22} - 0.15)} \text{ (Eqn. 17d.) where } \ddot{R} = -0.125 \frac{m}{s^2}$$

The acceleration data for point P during this move can be found in Appendix A.9. Figure 20 shows the relationship between the acceleration of point P in X and Y, and the input length of R_{22}. A_x begins positive and A_y begins negative, but at the end of the move A_x is negative and $A_y$is positive.

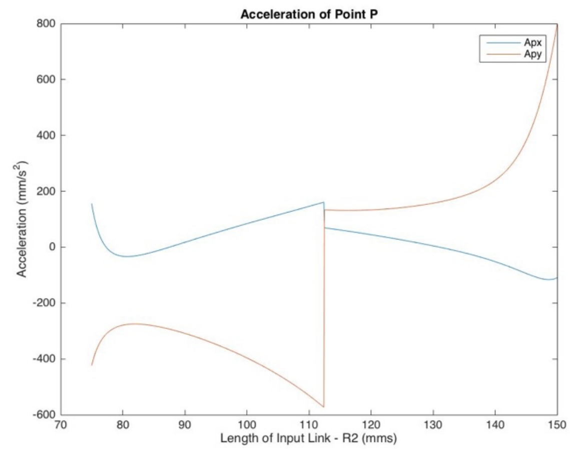

Figure 20: Acceleration of Point P

Instant Center Method Analysis

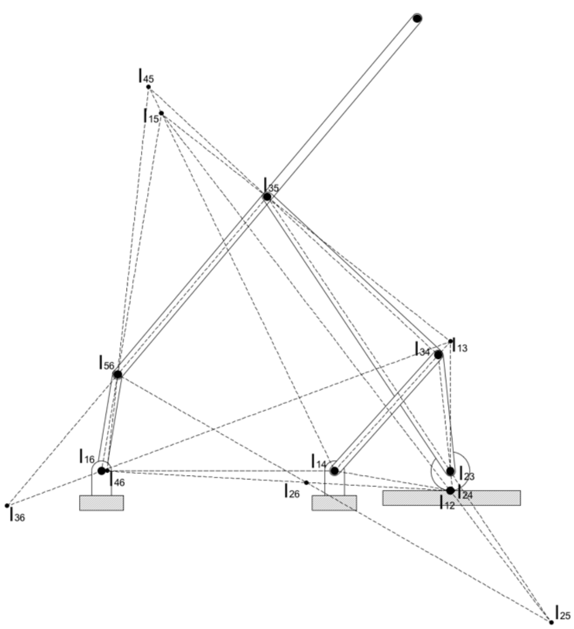

Figure 21: Instant Centers of Mechanism (Scale: 1”=2”)

Position Analysis:
In this six-bar mechanism, the number of instant centers can be determined using the following equation:

$N = \frac{n(n-1)}{2}$ (Eqn. 18) = $\frac{6(6-1)}{2}$=15
where n is the number of links, and N is the number of instant centers. This equation states that there must be 15 instant centers, and it can be easily determined that 7 of these 15 instant centers are primary. The primary instant centers are I_{12}, I_{14}, I_{16}, I_{23}, I_{34}, I_{35}, and I_{56}, and the secondary instant centers are I_{13}, I_{15}, I_{24}, I_{25}, I_{26}, I_{36}, I_{45}, and I_{46}.

Kinematic Coefficient Results:
To check the values of the First-Order Kinematic Coefficients, the following equations are used:

$\theta_2' = -0.08\, rad/mm$ $\quad \theta_3' = \frac{I_{12}I_{23}}{I_{13}I_{23}}$(Eqn. 18a.) $\quad \theta_4' = \frac{I_{12}I_{24}}{I_{14}I_{24}}$(Eqn. 18b.)
$\theta_5' = \frac{I_{12}I_{25}}{I_{15}I_{25}}$ (Eqn. 18c.) $\quad \theta_6' = \frac{I_{12}I_{26}}{I_{16}I_{26}}$(Eqn. 18d.)

From Appendix A.2:
θ_3'= 0.0119 rad/mm θ_4'= -0.0015 rad/mm θ_5'= -0.0205 rad/mm θ_6'= 0.0560 rad/mm.

The Instant Centers method calculates these first-order kinematic coefficients as:

$$\theta_3' = \frac{-I_{12}I_{23}}{I_{13}I_{23}}\theta_2' = \frac{-12.573}{83.6676} * -0.08 = 0.012\ \frac{mm}{mm} \quad \text{Error} = 0.83\%$$
$$\theta_4' = \frac{I_{12}I_{24}}{I_{14}I_{24}}\ \theta_2' = \frac{-1.3208}{76.9874} * -0.08 = -0.00137\ \frac{mm}{mm} \quad \text{Error} = 8.5\%$$
$$\theta_5' = \frac{I_{12}I_{25}}{I_{15}I_{25}}\ \theta_2' = \frac{107.0102}{413.1056} * -0.08 = -0.0207\ \frac{mm}{mm} \quad \text{Error} = 0.97\%.$$
$$\theta_6' = \frac{I_{12}I_{26}}{I_{16}I_{26}}\ \theta_2' = \frac{92.7608}{132.1816} * -0.08 = -0.0561\ \frac{mm}{mm} \quad \text{Error} = 0.18\%.$$

Once again the percentage error values are quite small. This is good, because it means that the Matlab program is calculating correctly because it agrees with the Instant Centers method.

Velocity Results:
It is necessary to find the distance from point P to the instant center of link 5. Using the Instant Center drawing (Figure 21.) this distance can be measured in its components.

Vp(From Analytical method) = 18.653i -207.02j= 207.86 mm/s
Vp(From the Instant Center method) = $I_{15}I_P$=164.64i+61.011j=175.6156 mm/s

Error = 15.51%

The error is still relatively small, and therefore it can be determined that velocity is being calculated correctly in the Matlab program.

Significance of Results:
The largest issue with the current design is that the equations for $\dot{R}\ and\ \ddot{R}$ flip in sign when R_{22} = 112.5 mm. This causes violent changes that are seen on plots above, and in application it would significantly reduce the mechanisms ability and efficiency. If the mechanism could be developed to have $\dot{R}\ and\ \ddot{R}$ equations such that one would fade nicely into the other, the mechanism would have a much smoother motion. By completing the analysis in Newton Raphson, the Instant Center method, and then using finite differences to check both of them, it suggests that calculations being completed by the Matlab program are correct. It is important to know how altering a variable can change other aspects of a design, and that altering a variable can affect a mechanisms movement greatly. Another check to verify results might be to build a small-scale prototype and take measurements to see if the mechanism will work as expected or use some sort of modeling software to complete a similar process. Over the course of this assignment I have learned a lot about how to work with systems that have a lot of components, and how to alter these links to help a machine work better. I also learned a lot about programming in Matlab. In real world machine design, all of these tools will be quite useful as they provide a basis to being questioning with when there is a new design needing to be created.

Appendix

A.1

R_{22}: (mm)	θ_3: ($degrees$)	θ_4: ($degrees$)	θ_5: ($degrees$)	θ_6: ($degrees$)
75.0	96.38	228.19	229.94	260.71
77.5	98.07	227.95	227.43	267.69
80.0	99.72	227.67	225.45	273.35
82.5	101.33	227.34	223.77	278.26
85.0	102.92	226.97	222.26	282.66
87.5	104.48	226.57	220.89	286.69
90.0	106.02	226.13	219.60	290.43
92.5	107.54	225.65	218.38	293.96
95.0	109.05	225.15	217.20	297.29
97.5	110.54	224.61	216.06	300.47
100.0	112.02	224.05	214.95	303.53
102.5	113.50	223.46	213.85	306.47
105.0	114.97	222.83	212.76	309.32
107.5	116.45	222.18	211.68	312.08
110.0	117.92	221.51	210.59	314.76
112.5	119.39	220.80	209.50	317.38
115.0	120.87	220.07	208.39	319.93
117.5	122.35	219.32	207.27	322.42
120.0	123.84	218.53	206.12	324.85
122.5	125.35	217.71	204.94	327.23
125.0	126.87	216.87	203.72	329.54
127.5	128.41	215.99	202.46	331.77
130.0	129.96	215.09	201.13	333.93
132.5	131.54	214.15	199.74	335.98
135.0	133.15	213.17	198.26	337.90
137.5	134.79	212.16	196.66	339.65
140.0	136.47	211.10	194.92	341.16
142.5	138.19	210.00	192.97	342.32
145.0	139.95	208.85	190.76	342.99
147.5	141.78	207.65	188.16	342.93
150.0	143.66	206.38	185.06	341.81

A.2

R_{22}: (mm)	θ_3': ($\frac{rad}{mm}$)	θ_4': ($\frac{rad}{mm}$)	θ_5': ($\frac{rad}{mm}$)	θ_6': ($\frac{rad}{mm}$)
75.0	0.0119	-0.0015	-0.0205	0.0560
77.5	0.0116	-0.0018	-0.0152	0.0432
80.0	0.0114	-0.0021	-0.0126	0.0365
82.5	0.0112	-0.0024	-0.0111	0.0323
85.0	0.0110	-0.0027	-0.0100	0.0293
87.5	0.0108	-0.0030	-0.0093	0.0271
90.0	0.0107	-0.0032	-0.0087	0.0253
92.5	0.0106	-0.0034	-0.0083	0.0239
95.0	0.0105	-0.0036	-0.0081	0.0227
97.5	0.0104	-0.0038	-0.0079	0.0217
100.0	0.0103	-0.0040	-0.0077	0.0209
102.5	0.0103	-0.0042	-0.0076	0.0202
105.0	0.0103	-0.0044	-0.0076	0.0196
107.5	0.0103	-0.0046	-0.0076	0.0190
110.0	0.0103	-0.0048	-0.0076	0.0185
112.5	0.0103	-0.0050	-0.0077	0.0180
115.0	0.0103	-0.0052	-0.0078	0.0176
117.5	0.0104	-0.0054	-0.0079	0.0172
120.0	0.0105	-0.0056	-0.0081	0.0168
122.5	0.0106	-0.0058	-0.0084	0.0163
125.0	0.0107	-0.0060	-0.0086	0.0159
127.5	0.0108	-0.0062	-0.0090	0.0154
130.0	0.0109	-0.0064	-0.0095	0.0147
132.5	0.0111	-0.0067	-0.0100	0.0139
135.0	0.0113	-0.0069	-0.0107	0.0129
137.5	0.0116	-0.0072	-0.0116	0.0115
140.0	0.0118	-0.0075	-0.0128	0.0095
142.5	0.0122	-0.0078	-0.0144	0.0066
145.0	0.0125	-0.0082	-0.0166	0.0024
147.5	0.0129	-0.0086	-0.0197	-0.0037
150.0	0.0134	-0.0091	-0.0239	-0.0125

A.3

R_{22}: (mm)	θ_3'': $(\frac{rad}{mm^2})$	θ_4'': $(\frac{rad}{mm^2})$	θ_5'': $(\frac{rad}{mm^2})$	θ_6'': $(\frac{rad}{mm^2})$
75.0	-0.00012	-0.00014	0.00321	-0.00776
77.5	-0.00011	-0.00013	0.00139	-0.00347
80.0	-0.00009	-0.00012	0.00079	-0.00206
82.5	-0.00008	-0.00011	0.00051	-0.00140
85.0	-0.00007	-0.00010	0.00035	-0.00102
87.5	-0.00006	-0.00010	0.00025	-0.00079
90.0	-0.00005	-0.00009	0.00018	-0.00063
92.5	-0.00004	-0.00009	0.00013	-0.00051
95.0	-0.00003	-0.00009	0.00010	-0.00043
97.5	-0.00003	-0.00008	0.00007	-0.00036
100.0	-0.00002	-0.00008	0.00005	-0.00031
102.5	-0.00001	-0.00008	0.00003	-0.00027
105.0	-0.00001	-0.00008	0.00001	-0.00024
107.5	-0.00000	-0.00008	-0.00001	-0.00021
110.0	0.00001	-0.00008	-0.00002	-0.00019
112.5	0.00001	-0.00008	-0.00004	-0.00018
115.0	0.00002	-0.00008	-0.00005	-0.00017
117.5	0.00003	-0.00008	-0.00007	-0.00016
120.0	0.00003	-0.00008	-0.00008	-0.00017
122.5	0.00004	-0.00008	-0.00010	-0.00018
125.0	0.00005	-0.00009	-0.00013	-0.00020
127.5	0.00006	-0.00009	-0.00016	-0.00023
130.0	0.00007	-0.00009	-0.00020	-0.00028
132.5	0.00008	-0.00010	-0.00025	-0.00036
135.0	0.00009	-0.00011	-0.00032	-0.00048
137.5	0.00010	-0.00011	-0.00041	-0.00067
140.0	0.00012	-0.00012	-0.00055	-0.00094
142.5	0.00013	-0.00014	-0.00075	-0.00137
145.0	0.00016	-0.00015	-0.00103	-0.00201
147.5	0.00018	-0.00017	-0.00143	-0.00294
150.0	0.00022	-0.00019	-0.00192	-0.00416

A.4

R_{22}: *(mm)*	ω_2: *(rad/s)*	ω_3: *(rad/s)*	ω_4: *(rad/s)*	ω_5: *(rad/s)*	ω_6: *(rad/s)*
75.0	-0.000	0.000	-0.000	-0.000	0.000
77.5	-2.000	0.291	-0.046	-0.381	1.079
80.0	-2.828	0.403	-0.076	-0.447	1.291
82.5	-3.464	0.484	-0.105	-0.479	1.398
85.0	-4.000	0.549	-0.135	-0.500	1.465
87.5	-4.472	0.605	-0.165	-0.518	1.513
90.0	-4.899	0.654	-0.195	-0.535	1.550
92.5	-5.292	0.699	-0.226	-0.552	1.581
95.0	-5.657	0.740	-0.257	-0.570	1.607
97.5	-6.000	0.780	-0.288	-0.589	1.631
100.0	-6.325	0.817	-0.320	-0.610	1.653
102.5	-6.633	0.854	-0.352	-0.632	1.674
105.0	-6.928	0.890	-0.384	-0.656	1.694
107.5	-7.211	0.925	-0.417	-0.683	1.713
110.0	-7.483	0.961	-0.451	-0.712	1.730
112.5	-7.746	0.997	-0.485	-0.744	1.746
115.0	-7.483	0.967	-0.486	-0.728	1.647
117.5	-7.211	0.937	-0.486	-0.715	1.549
120.0	-6.928	0.906	-0.484	-0.703	1.453
122.5	-6.633	0.875	-0.480	-0.693	1.356
125.0	-6.325	0.843	-0.474	-0.684	1.256
127.5	-6.000	0.810	-0.466	-0.676	1.151
130.0	-5.657	0.774	-0.456	-0.668	1.041
132.5	-5.292	0.736	-0.442	-0.662	0.920
135.0	-4.899	0.694	-0.425	-0.656	0.788
137.5	-4.472	0.647	-0.404	-0.650	0.640
140.0	-4.000	0.592	-0.376	-0.641	0.473
142.5	-3.464	0.526	-0.340	-0.625	0.286
145.0	-2.828	0.443	-0.290	-0.588	0.086
147.5	-2.000	0.324	-0.215	-0.492	-0.092
150.0	-0.000	0.000	-0.000	-0.000	-0.000

A.5

R_{22}: (mm)	α_2: $\left(\frac{rad}{s^2}\right)$	α_3: $\left(\frac{rad}{s^2}\right)$	α_4: $\left(\frac{rad}{s^2}\right)$	α_5: $\left(\frac{rad}{s^2}\right)$	α_6: $\left(\frac{rad}{s^2}\right)$
75.0	-10.00	1.491	-0.186	-2.560	7.003
77.5	-10.00	1.388	-0.310	-1.039	3.228
80.0	-10.00	1.307	-0.417	-0.596	1.991
82.5	-10.00	1.244	-0.512	-0.435	1.419
85.0	-10.00	1.198	-0.598	-0.382	1.109
87.5	-10.00	1.166	-0.676	-0.383	0.927
90.0	-10.00	1.146	-0.748	-0.412	0.817
92.5	-10.00	1.137	-0.815	-0.461	0.750
95.0	-10.00	1.138	-0.880	-0.522	0.711
97.5	-10.00	1.149	-0.943	-0.595	0.689
100.0	-10.00	1.169	-1.005	-0.678	0.678
102.5	-10.00	1.198	-1.067	-0.771	0.672
105.0	-10.00	1.236	-1.131	-0.877	0.666
107.5	-10.00	1.283	-1.196	-0.997	0.655
110.0	-10.00	1.339	-1.265	-1.133	0.632
112.5	10.00	-1.168	-0.087	0.628	-3.919
115.0	10.00	-1.124	-0.022	0.533	-3.673
117.5	10.00	-1.089	0.040	0.452	-3.486
120.0	10.00	-1.062	0.100	0.383	-3.350
122.5	10.00	-1.044	0.158	0.324	-3.261
125.0	10.00	-1.033	0.217	0.272	-3.217
127.5	10.00	-1.032	0.276	0.227	-3.215
130.0	10.00	-1.040	0.337	0.189	-3.253
132.5	10.00	-1.057	0.401	0.160	-3.325
135.0	10.00	-1.087	0.470	0.146	-3.419
137.5	10.00	-1.130	0.545	0.160	-3.510
140.0	10.00	-1.188	0.628	0.227	-3.542
142.5	10.00	-1.266	0.724	0.402	-3.392
145.0	10.00	-1.369	0.836	0.789	-2.817
147.5	10.00	-1.503	0.969	1.567	-1.377
150.0	10.00	-1.680	1.133	2.982	1.564

A.6

R_{22}: (mm)	Xp: (mm)	Yp: (mm)	Xp': $(\frac{mm}{mm})$	Yp': $(\frac{mm}{mm})$	Xp": $(\frac{mm}{mm^2})$	Yp": $(\frac{mm}{mm^2})$
75.0	203.17	291.29	1.25	-3.39	-0.370	0.251
77.5	205.46	283.39	0.67	-2.98	-0.141	0.105
80.0	206.79	276.20	0.42	-2.79	-0.068	0.056
82.5	207.67	269.37	0.30	-2.69	-0.036	0.032
85.0	208.32	262.74	0.23	-2.62	-0.018	0.019
87.5	208.86	256.23	0.20	-2.59	-0.008	0.010
90.0	209.34	249.79	0.19	-2.57	-0.002	0.003
92.5	209.81	243.36	0.19	-2.57	0.002	-0.001
95.0	210.30	236.93	0.20	-2.58	0.005	-0.005
97.5	210.81	230.47	0.22	-2.59	0.007	-0.008
100.0	211.38	223.96	0.24	-2.62	0.009	-0.011
102.5	211.99	217.37	0.26	-2.65	0.010	-0.014
105.0	212.67	210.71	0.29	-2.69	0.011	-0.016
107.5	213.42	203.94	0.31	-2.73	0.011	-0.018
110.0	214.23	197.06	0.34	-2.78	0.012	-0.021
112.5	215.12	190.04	0.37	-2.84	0.012	-0.024
115.0	216.09	182.88	0.40	-2.90	0.013	-0.026
117.5	217.14	175.55	0.44	-2.97	0.013	-0.029
120.0	218.26	168.04	0.47	-3.05	0.014	-0.033
122.5	219.48	160.31	0.50	-3.13	0.014	-0.037
125.0	220.79	152.36	0.54	-3.23	0.015	-0.042
127.5	222.18	144.15	0.58	-3.34	0.016	-0.048
130.0	223.68	135.64	0.62	-3.47	0.016	-0.055
132.5	225.28	126.78	0.66	-3.62	0.017	-0.065
135.0	226.98	117.52	0.71	-3.80	0.018	-0.077
137.5	228.80	107.77	0.75	-4.01	0.019	-0.094
140.0	230.74	97.41	0.80	-4.28	0.019	-0.118
142.5	232.80	86.32	0.85	-4.61	0.018	-0.152
145.0	234.96	74.28	0.89	-5.05	0.013	-0.199
147.5	237.21	60.95	0.91	-5.62	0.001	-0.265
150.0	239.45	45.97	0.88	-6.39	0.029	-0.346

A.7

R_{22}: (mm)	Ut_x: $(None)$	Ut_y: $(None)$	Un_x: $(None)$	Un_y: $(None)$
75.0	0.345	-0.939	0.939	0.345
77.5	0.219	-0.976	0.976	0.219
80.0	0.150	-0.989	0.989	0.150
82.5	0.110	-0.994	0.994	0.110
85.0	0.088	-0.996	0.996	0.088
87.5	0.077	-0.997	0.997	0.077
90.0	0.073	-0.997	0.997	0.073
92.5	0.074	-0.997	0.997	0.074
95.0	0.077	-0.997	0.997	0.077
97.5	0.083	-0.997	0.997	0.083
100.0	0.090	-0.996	0.996	0.090
102.5	0.097	-0.995	0.995	0.097
105.0	0.105	-0.994	0.994	0.105
107.5	0.114	-0.994	0.994	0.114
110.0	0.122	-0.993	0.993	0.122
112.5	0.130	-0.992	0.992	0.130
115.0	0.138	-0.990	0.990	0.138
117.5	0.145	-0.989	0.989	0.145
120.0	0.152	-0.988	0.988	0.152
122.5	0.159	-0.987	0.987	0.159
125.0	0.165	-0.986	0.986	0.165
127.5	0.170	-0.985	0.985	0.170
130.0	0.175	-0.985	0.985	0.175
132.5	0.179	-0.984	0.984	0.179
135.0	0.182	-0.983	0.983	0.182
137.5	0.184	-0.983	0.983	0.184
140.0	0.184	-0.983	0.983	0.184
142.5	0.180	-0.984	0.984	0.180
145.0	0.173	-0.985	0.985	0.173
147.5	0.159	-0.987	0.987	0.159
150.0	0.136	-0.991	0.991	0.136

A.8

R_{22}: (mm)	ρ: (mm)	X_{cc}: (mm)	Y_{cc}: (mm)
75.0	-50.01	156.24	274.02
77.5	-81.38	126.06	265.54
80.0	-134.47	73.83	256.08
82.5	-228.87	-19.81	244.19
85.0	-419.10	-209.15	225.83
87.5	-924.80	-713.19	184.96
90.0	-4648.20	-4426.43	-89.99
92.5	2778.74	2981.00	448.08
95.0	1359.73	1565.96	342.02
97.5	1027.44	1234.72	315.62
100.0	897.16	1104.92	304.46
102.5	841.89	1049.88	299.32
105.0	824.62	1032.70	297.60
107.5	831.07	1039.11	298.32
110.0	855.03	1062.91	301.16
112.5	894.00	1101.56	306.07
115.0	947.64	1154.72	313.26
117.5	1017.39	1223.76	323.13
120.0	1106.73	1312.11	336.43
122.5	1222.09	1426.07	354.35
125.0	1376.27	1578.21	379.34
127.5	1593.73	1792.58	415.86
130.0	1929.58	2123.36	474.03
132.5	2534.37	2718.52	581.50
135.0	4007.47	4167.20	848.63
137.5	13705.02	13699.75	2629.90
140.0	-6670.72	-6326.51	-1127.75
142.5	-2281.53	-2011.28	-325.37
145.0	-1231.31	-977.79	-138.72
147.5	-780.40	-533.25	-63.19
150.0	-550.09	-305.54	-28.74

A.9

R_{22}: (mm)	Vp_x: $(\frac{mm}{s})$	Vp_y: $(\frac{mm}{s})$	Ap_x: $(\frac{mm}{s^2})$	Ap_y: $(\frac{mm}{s^2})$
75.0	0.000	-0.000	155.788	-423.571
77.5	16.774	-74.590	-4.476	-307.449
80.0	14.935	-98.696	-32.683	-279.274
82.5	12.866	-116.251	-29.670	-275.281
85.0	11.594	-131.134	-16.631	-281.471
87.5	11.185	-144.679	-0.172	-293.200
90.0	11.543	-157.507	17.240	-308.587
92.5	12.559	-169.965	34.618	-326.809
95.0	14.133	-182.272	51.590	-347.529
97.5	16.185	-194.584	68.057	-370.676
100.0	18.653	-207.021	84.057	-396.345
102.5	21.488	-219.684	99.692	-424.765
105.0	24.655	-232.666	115.103	-456.285
107.5	28.128	-246.058	130.453	-491.386
110.0	31.888	-259.956	145.923	-530.705
112.5	35.926	-274.463	68.951	133.578
115.0	37.641	-270.986	61.471	131.557
117.5	39.204	-267.410	53.287	131.504
120.0	40.588	-263.669	44.558	133.221
122.5	41.767	-259.694	35.362	136.604
125.0	42.710	-255.403	25.706	141.642
127.5	43.379	-250.699	15.528	148.452
130.0	43.726	-245.458	4.689	157.342
132.5	43.684	-239.505	-7.044	168.941
135.0	43.154	-232.580	-20.026	184.478
137.5	41.987	-224.261	-34.765	206.328

```
%ME 352 - LAB PROJECT 2 - SIX-BAR LINKAGE - ELENA HELVAJIAN
clear
clc

%GIVEN VALUES
    %LENGTH
    R1 = 150; %mms
    R3 = 75; %mms
    R33 = 212.5; %mms
    R4 = 100; %mms
    R5 = 150; %mms
    R6 = 62.5; %mms
    BC = 150; %mms
    CP = 150; %mms
    rho = 12.5; %mms

    %ANGLE
    alpha = acos((150^2+212.5^2-75^2)/(2*212.5*150))*180/pi; %rads (Angle between AC and↙
BC)
    beta = acos((150^2+75^2-212.5^2)/(2*150*75)); %rads (Angle between AB and BC)
    theta_1 = 0; %rads
    theta_2 = 0; %rads
    theta_22 = 0; %rads
    theta_3_guess = 96.38/180*pi; %rads (original guess for Theta 3)
    theta_4_guess = 228.19/180*pi; %rads (original guess for Theta 4)
    theta_5_guess = 229.94/180*pi; %rads (original guess for Theta 5)
    theta_6_guess = 260.7/180*pi; %rads (original guess for Theta 6)

    %ADDITIONAL INFORMATION
    tol = 0.01/180*pi; %tolerance in radians

    %ACCELERATION
    R_doubledot_22 = 0.125; %m/s^2

    %INITIALIZE VECTORS
    R22all = []; %mms
    theta_3all = []; %rads
    theta_4all = []; %rads
    theta_5all = []; %rads
    theta_6all = []; %rads
    theta_3_primeall = []; %mm/mm
    theta_4_primeall = []; %mm/mm
    theta_5_primeall = []; %mm/mm
    theta_6_primeall = []; %mm/mm
    theta_3_2primeall = []; %mm/mm^2
    theta_4_2primeall = []; %mm/mm^2
    theta_5_2primeall = []; %mm/mm^2
    theta_6_2primeall = []; %mm/mm^2
    omega_2all = []; %rad/s
    omega_3all = []; %rad/s
    omega_4all = []; %rad/s
    omega_5all = []; %rad/s
    omega_6all = []; %rad/s
    accel_2all = []; %rad/s^2
    accel_3all = []; %rad/s^2
    accel_4all = []; %rad/s^2
    accel_5all = []; %rad/s^2
    accel_6all = []; %rad/s^2
```

```
    Xp_all = []; %mms
    Yp_all = []; %mms
    Xp_primeall = []; %mm/mm
    Yp_primeall = []; %mm/mm
    Xp_2primeall = []; %mm/mm^2
    Yp_2primeall = []; %mm/mm^2
    Ut_X_all = []; %no units
    Ut_Y_all = []; %no units
    Un_X_all = []; %no units
    Un_Y_all = []; %no units
    rho_c_all = []; %mms
    Xcc_all = []; %mms
    Ycc_all = []; %mms
    Vp_X_all = []; %mm/s
    Vp_Y_all = []; %mm/s
    Ap_X_all = []; %mm/s^2
    Ap_Y_all = []; %mm/s^2
    theta_2primeall = []; %mm/mm

%CALCULATIONS
for R22 = 75:0.1:150
    R22all = [R22all,R22];  %mms (All R22 values in this vector)

    % CALCULATE THETA 3 AND 4 USING NEWTON RAPHSON
    ex = R4*cos(theta_4_guess)+R3*cos(theta_3_guess) + R22*cos(theta_22);
    ey = R4*sin(theta_4_guess)+R3*sin(theta_3_guess) + R22*sin(theta_22);
    a11 = -R3*sin(theta_3_guess);
    a12 = -R4*sin(theta_4_guess);
    a21 = R3*cos(theta_3_guess);
    a22 = R4*cos(theta_4_guess);

    determinant1 = (a11*a22)-(a21*a12);
    delta_theta_3 = (-ex*a22-(-ey)*a12)/determinant1; %value the guess of Theta 3 needs↙
to change by
    delta_theta_4 = (-ey*a11-(-ex)*a21)/determinant1; %value the guess of Theta 4 needs↙
to change by

    while abs(delta_theta_3) > tol || abs(delta_theta_4) > tol
        ex = R4*cos(theta_4_guess)+R3*cos(theta_3_guess) + R22*cos(theta_22);
        ey = R4*sin(theta_4_guess)+R3*sin(theta_3_guess) + R22*sin(theta_22);
        a11 = -R3*sin(theta_3_guess);
        a12 = -R4*sin(theta_4_guess);
        a21 = R3*cos(theta_3_guess);
        a22 = R4*cos(theta_4_guess);
        determinant1 = (a11*a22)-(a12*a21);

        delta_theta_3 = ((-ex*a22)-((-ey)*a12))/determinant1; %value the guess of Theta 3↙
needs to change by
        delta_theta_4 = ((-ey*a11)-((-ex)*a21))/determinant1; %value the guess of Theta 4↙
needs to change by
        theta_3_guess = (theta_3_guess + delta_theta_3); %rads (final value of Theta 3)
        theta_4_guess = (theta_4_guess + delta_theta_4); %rads (final value of Theta 4)
    end

    theta_3all = [theta_3all, theta_3_guess*180/pi]; %rads (All Theta 3 values in this↙
vector)
    theta_4all = [theta_4all, theta_4_guess*180/pi]; %rads (All Theta 4 values in this↙
vector)
```

```
    %CALCULATE FIRST-ORDER KINEMATIC COEFFICIENT FOR THETA 3 AND 4
    b11 = -R3*sin(theta_3_guess);
    b12 = -R4*sin(theta_4_guess);
    b21 = R3*cos(theta_3_guess);
    b22 = R4*cos(theta_4_guess);

    val1 = -cos(theta_22);
    val2 = -sin(theta_22);
    determinant2 = (b11*b22)-(b12*b21);

    theta_3_prime = ((val1 * b22) - (b12 * val2))/determinant2; %mm/mm
    theta_4_prime = ((b11 * val2) - (val1 * b21))/determinant2; %mm/mm
    theta_3_primeall = [theta_3_primeall,theta_3_prime]; %mm/mm (All Theta 3 Prime values↙
in this vector)
    theta_4_primeall = [theta_4_primeall,theta_4_prime]; %mm/mm (All Theta 4 Prime values↙
in this vector)

    %CALCULATE SECOND-ORDER KINEMATIC COEFFICIENT FOR THETA 3 AND 4
    val3 = R3*cos(theta_3_guess)*(theta_3_prime^2) + R4*cos(theta_4_guess)*↙
(theta_4_prime^2);
    val4 = R3*sin(theta_3_guess)*(theta_3_prime^2) + R4*sin(theta_4_guess)*↙
(theta_4_prime^2);
    theta_3_2prime = ((val3 * b22) - (b12 * val4)) / determinant2; %mm/mm^2
    theta_4_2prime = ((b11 * val4) - (val3 * b21)) / determinant2; %mm/mm^2
    theta_3_2primeall = [theta_3_2primeall,theta_3_2prime]; %mm/mm^2 (All Theta 3 Double↙
Prime values in this vector)
    theta_4_2primeall = [theta_4_2primeall,theta_4_2prime]; %mm/mm^2 (All Theta 4 Double↙
Prime values in this vector)

    %CONSTRAINT FOR VLE #2 - UPDATE THE VALUE OF THETA 33
    theta_33 = theta_3_guess + (27.47)/180*pi;

    %CALCULATE THETA 5 AND 6 USING NEWTON RAPHSON
    ex2 = R22*cos(theta_22)+R33*cos(theta_33)+R5*cos(theta_5_guess)+R6*cos(theta_6_guess)↙
+R1*cos(theta_1);
    ey2 = R22*sin(theta_22)+R33*sin(theta_33)+R5*sin(theta_5_guess)+R6*sin(theta_6_guess)↙
+R1*sin(theta_1);
    c11 = -R5*sin(theta_5_guess);
    c12 = -R6*sin(theta_6_guess);
    c21 = R5*cos(theta_5_guess);
    c22 = R6*cos(theta_6_guess);

    determinant3 = (c11*c22)-(c12*c21);
    delta_theta_5 = (((-ex2)*c22)-((-ey2)*c12))/determinant3; %value the guess of Theta 5↙
needs to change by
    delta_theta_6 = (((-ey2)*c11)-((-ex2)*c21))/determinant3; %value the guess of Theta 6↙
needs to change by

    while abs(delta_theta_5) > tol || abs(delta_theta_6) > tol
        ex2 = R22*cos(theta_22)+R33*cos(theta_33)+R5*cos(theta_5_guess)+R6*cos↙
(theta_6_guess)+R1*cos(theta_1);
        ey2 = R22*sin(theta_22)+R33*sin(theta_33)+R5*sin(theta_5_guess)+R6*sin↙
(theta_6_guess)+R1*sin(theta_1);
        c11 = -R5*sin(theta_5_guess);
        c12 = -R6*sin(theta_6_guess);
        c21 = R5*cos(theta_5_guess);
        c22 = R5*cos(theta_6_guess);
```

```
        determinant3 = (c11*c22)-(c12*c21);
        delta_theta_5 = (((-ex2)*c22)-((-ey2)*c12))/determinant3; %value the guess of↙
Theta 5 needs to change by
        delta_theta_6 = (((-ey2)*c11)-((-ex2)*c21))/determinant3; %value the guess of↙
Theta 6 needs to change by
        theta_5_guess = theta_5_guess + delta_theta_5; %rads (final value of Theta 5)
        theta_6_guess = theta_6_guess + delta_theta_6; %rads (final value of Theta 6)
    end

    theta_5all = [theta_5all, theta_5_guess*180/pi]; %rads (All Theta 5 values in this↙
vector)
    theta_6all = [theta_6all, theta_6_guess*180/pi]; %rads (All Theta 6 values in this↙
vector)

    %CALCULATE FIRST-ORDER KINEMATIC COEFFICIENT FOR THETA 5 AND 6
    d11 = -R5*sin(theta_5_guess);
    d12 = -R6*sin(theta_6_guess);
    d21 = R5*cos(theta_5_guess);
    d22 = R6*cos(theta_6_guess);

    val5 = -cos(theta_22) + R33*sin(theta_33)*theta_3_prime;
    val6 = -sin(theta_22) - R33*cos(theta_33)*theta_3_prime;
    determinant4 = (d11*d22) - (d12*d21);

    theta_5_prime = ((val5 * d22) - (d12 * val6)) / determinant4; %mm/mm
    theta_6_prime = ((d11 * val6) - (val5 * d21)) / determinant4; %mm/mm
    theta_5_primeall = [theta_5_primeall,theta_5_prime]; %mm/mm (All Theta 5 Prime values↙
in this vector)
    theta_6_primeall = [theta_6_primeall,theta_6_prime]; %mm/mm (All Theta 6 Prime values↙
in this vector)

    %CALCULATE SECOND ORDER KINEMATIC COEFFICIENTS FOR THETA 3 AND 4
    val7 = R33*cos(theta_33)*(theta_3_prime^2)+R33*sin(theta_33)*(theta_3_2prime)+R5*cos↙
(theta_5_guess)*(theta_5_prime^2)+R6*cos(theta_6_guess)*(theta_6_prime^2);
    val8 = R33*sin(theta_33)*(theta_3_prime^2)-R33*cos(theta_33)*(theta_3_2prime)+R5*sin↙
(theta_5_guess)*(theta_5_prime^2)+R6*sin(theta_6_guess)*(theta_6_prime^2);
    theta_5_2prime = ((val7 * d22) - (d12 * val8)) / determinant4; %mm/mm^2
    theta_6_2prime = ((d11 * val8) - (val7 * d21)) / determinant4; %mm/mm^2
    theta_5_2primeall = [theta_5_2primeall,theta_5_2prime]; %mm/mm^2 (All Theta 5 Double↙
Prime values in this vector)
    theta_6_2primeall = [theta_6_2primeall,theta_6_2prime]; %mm/mm^2 (All Theta 6 Double↙
Prime values in this vector)

    %CALCULATE OMEGA 2, 3, 4, 5, AND 6
    if R22 < 112.5 %mm
        %Angular Velocity of all links when R22 is less than 112.5 mm
        omega_2 = -sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000/rho;
        omega_3 = theta_3_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000;
        omega_4 = theta_4_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000;
        omega_5 = theta_5_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000;
        omega_6 = theta_6_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000;
        %Angular Acceleration of all links when R22 is less than 112.5 mm
        accel_2 = -R_doubledot_22*1000/rho;
        accel_3 = theta_3_2prime*(sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000)↙
^2+theta_3_prime*R_doubledot_22*1000;
        accel_4 = theta_4_2prime*(sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000)↙
^2+theta_4_prime*R_doubledot_22*1000;
```

```
        accel_5 = theta_5_2prime*(sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000)↙
^2+theta_5_prime*R_doubledot_22*1000;
        accel_6 = theta_6_2prime*(sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000)↙
^2+theta_6_prime*R_doubledot_22*1000;

        %Update Vectors
        omega_2all = [omega_2all,omega_2]; %rad/s
        omega_3all = [omega_3all,omega_3]; %rad/s
        omega_4all = [omega_4all,omega_4]; %rad/s
        omega_5all = [omega_5all,omega_5]; %rad/s
        omega_6all = [omega_6all,omega_6]; %rad/s
        accel_2all = [accel_2all,accel_2]; %rad/s^2
        accel_3all = [accel_3all,accel_3]; %rad/s^2
        accel_4all = [accel_4all,accel_4]; %rad/s^2
        accel_5all = [accel_5all,accel_5]; %rad/s^2
        accel_6all = [accel_6all,accel_6]; %rad/s^2
    else
        %Angular Velocity of all links when R22 is greater than or equal to 112.5 mm
        omega_2 = -sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000/rho;
        omega_3 = theta_3_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000;
        omega_4 = theta_4_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000;
        omega_5 = theta_5_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000;
        omega_6 = theta_6_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000;
        %Angular Acceleration of all links when R22 is greater than or equal to 112.5 mm
        accel_2 = R_doubledot_22*1000/rho;
        accel_3 = theta_3_2prime*(sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000)↙
^2+theta_3_prime*-R_doubledot_22*1000;
        accel_4 = theta_4_2prime*(sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000)↙
^2+theta_4_prime*-R_doubledot_22*1000;
        accel_5 = theta_5_2prime*(sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000)↙
^2+theta_5_prime*-R_doubledot_22*1000;
        accel_6 = theta_6_2prime*(sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000)↙
^2+theta_6_prime*-R_doubledot_22*1000;

        %Update Vectors
        omega_2all = [omega_2all,omega_2]; %rad/s
        omega_3all = [omega_3all,omega_3]; %rad/s
        omega_4all = [omega_4all,omega_4]; %rad/s
        omega_5all = [omega_5all,omega_5]; %rad/s
        omega_6all = [omega_6all,omega_6]; %rad/s
        accel_2all = [accel_2all,accel_2]; %rad/s^2
        accel_3all = [accel_3all,accel_3]; %rad/s^2
        accel_4all = [accel_4all,accel_4]; %rad/s^2
        accel_5all = [accel_5all,accel_5]; %rad/s^2
        accel_6all = [accel_6all,accel_6]; %rad/s^2
    end

    %ANALYSIS OF POINT P
        %POSITION OF P
        Xp = -2*R5*cos(theta_5_guess)-R6*cos(theta_6_guess); %position of point P in the↙
X direction (mms)
        Yp = -2*R5*sin(theta_5_guess)-R6*sin(theta_6_guess); %position of point P in the↙
Y direction (mms)

        %FIRST-ORDER KINEMATIC COEFFICIENT
        Xp_prime = 2*R5*sin(theta_5_guess)*theta_5_prime+R6*sin(theta_6_guess)↙
*theta_6_prime;
        Yp_prime = -2*R5*cos(theta_5_guess)*theta_5_prime-R6*cos(theta_6_guess)↙
```

```
*theta_6_prime;

        %SECOND-ORDER KINEMATIC COEFFICIENT
        Xp_2prime = 2*R5*cos(theta_5_guess)*((theta_5_prime)^2)+2*R5*sin(theta_5_guess)↙
*theta_5_2prime+R6*cos(theta_6_guess)*((theta_6_prime)^2)+R6*sin(theta_6_guess)*↙
(theta_6_2prime);
        Yp_2prime = 2*R5*sin(theta_5_guess)*((theta_5_prime)^2)-2*R5*cos(theta_5_guess)↙
*theta_5_2prime+R6*sin(theta_6_guess)*((theta_6_prime)^2)-R6*cos(theta_6_guess)↙
*theta_6_2prime;

        %CALCULATE RADIUS OF CURVATURE AND CENTER OF CURVATURE
        Rp_prime = sqrt(Xp_prime^2+Yp_prime^2);
        rho_c = Rp_prime^3/(Xp_prime*Yp_2prime- Xp_2prime*Yp_prime);
        Ut_X = Xp_prime/Rp_prime; %Unit Tangent in the X direction
        Ut_Y = Yp_prime/Rp_prime; %Unit Tangent in the Y direction
        Un_X = -Yp_prime/Rp_prime; %Unit Normal in the X direction
        Un_Y = Xp_prime/Rp_prime; %Unit Normal in the Y direction
        Xcc = Xp + rho_c*Un_X;
        Ycc = Yp + rho_c*Un_Y;

    %Update Vectors
    Xp_all = [Xp_all,Xp]; %mms (All Xp values in this vector)
    Yp_all = [Yp_all,Yp]; %mms (All Yp values in this vector)
    Xp_primeall = [Xp_primeall,Xp_prime]; %mm/mm (All Xp Prime values in this vector)
    Yp_primeall = [Yp_primeall,Yp_prime]; %mm/mm (All Yp Prime values in this vector)
    Xp_2primeall = [Xp_2primeall,Xp_2prime]; %mm/mm^2 (All Xp Double Prime values in this↙
vector)
    Yp_2primeall = [Yp_2primeall,Yp_2prime]; %mm/mm^2 (All Yp Double Prime values in this↙
vector)
    Ut_X_all = [Ut_X_all,Ut_X]; %no units (Unit Tangent in the X direction)
    Un_X_all = [Un_X_all,Un_X]; %no units (Unit Tangent in the Y direction)
    Ut_Y_all = [Ut_Y_all,Ut_Y]; %no units (Unit Normal in the X direction)
    Un_Y_all = [Un_Y_all,Un_Y]; %no units (Unit Normal in the Y direction)
    rho_c_all = [rho_c_all,rho_c]; %mms (All Radius of Curvature values in this vector)
    Xcc_all = [Xcc_all,Xcc]; %mms (All Center of Curvature position values in the X↙
direction in this vector)
    Ycc_all = [Ycc_all,Ycc]; %mms (All Center of Curvature position values in the Y↙
direction in this vector)

    if R22 < 112.5
        Vp_X = Xp_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000; %mm/s (Velocity of↙
Point P in the X direction when R22 is greater than 112.5 mm)
        Vp_Y = Yp_prime*sqrt(2*R_doubledot_22*(R22/1000-0.075))*1000; %mm/s (Velocity of↙
Point P in the Y direction when R22 is greater than 112.5 mm)
        r_dot = sqrt(2*R_doubledot_22/100*(R22/1000-0.075))*1000;
        Ap_X = 100*Xp_2prime*(r_dot)^2+Xp_prime*R_doubledot_22*1000; %mm/s^2↙
(Acceleration of Point P in the X direction when R22 is greater than 112.5 mm)
        Ap_Y = 100*Yp_2prime*(r_dot)^2+Yp_prime*R_doubledot_22*1000; %mm/s^2↙
(Acceleration of Point P in the Y direction when R22 is greater than 112.5 mm)

    else
        Vp_X = Xp_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000; %mm/s (Velocity of↙
Point P in the X direction when R22 is greater than 112.5 mm)
        Vp_Y = Yp_prime*sqrt(2*-R_doubledot_22*(R22/1000-0.15))*1000; %mm/s (Velocity of↙
Point P in the Y direction when R22 is greater than 112.5 mm)
        r_dot = sqrt(2*-R_doubledot_22/100*(R22/1000-0.15))*1000;
        Ap_X = 100*Xp_2prime*(r_dot)^2+Xp_prime*-R_doubledot_22*1000; %mm/s^2↙
(Acceleration of Point P in the X direction when R22 is greater than 112.5 mm)
```

```
        Ap_Y = 100*Yp_2prime*(r_dot)^2+Yp_prime*-R_doubledot_22*1000; %mm/s^2↙
(Acceleration of Point P in the Y direction when R22 is greater than 112.5 mm)

    end
%Update Vectors
Vp_X_all = [Vp_X_all,Vp_X]; %mm/s (Velocity of Point P in the X direction)
Vp_Y_all = [Vp_Y_all,Vp_Y]; %mm/s (Velocity of Point P in the Y direction)
Ap_X_all = [Ap_X_all,Ap_X]; %mm/s^2 (Acceleration of Point P in the X direction)
Ap_Y_all = [Ap_Y_all,Ap_Y]; %mm/s^2 (Acceleration of Point P in the Y direction)

theta_2_prime = -1 / rho; %(rad/mm)
theta_2primeall = [theta_2primeall,theta_2_prime];
end
%

% %PRINTED DATA
    %TABULATED DATA FOR THETA 3,4,5,AND 6
    fprintf(' R22:   Theta 3:   Theta 4:   Theta 5:   Theta 6: \n')
    fprintf(' (mm)  (degrees)  (degrees)  (degrees)  (degrees) \n')
    for x=1:25:751
        fprintf('%5.1f    %6.2f     %6.2f     %6.2f     %6.2f \n',R22all(x),theta_3all↙
(x),theta_4all(x),theta_5all(x),theta_6all(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR THETA 3',4',5',AND 6'
    fprintf(' R22   Theta 2'':   Theta 3'':   Theta 4'':   Theta 5'':   Theta 6'':   \n')
    fprintf(' (mm)  (rad/mm)    (rad/mm)    (rad/mm)    (rad/mm)    (rad/mm)    \n')
    for x = 1:25:751
        fprintf('%5.1f   %6.2f      %6.4f      %6.4f     %6.4f     %6.4f \n',R22all(x),↙
theta_2primeall(x),theta_3_primeall(x),theta_4_primeall(x),theta_5_primeall(x),↙
theta_6_primeall(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR THETA 3",4",5",AND 6"
    fprintf(' R22       Theta 3":       Theta 4":       Theta 5":       Theta 6":\n')
    fprintf(' (mm)      (rad/mm^2)      (rad/mm^2)      (rad/mm^2)      (rad/mm^2)\n')
    for x = 1:25:751
        fprintf('%5.1f   %6.9f    %6.9f    %6.9f     %6.9f  \n ',R22all(x),↙
theta_3_2primeall(x),theta_4_2primeall(x),theta_5_2primeall(x),theta_6_2primeall(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR OMEGA 2,3,4,5,AND 6
    fprintf(' R22    Omega 2:  Omega 3:  Omega 4:  Omega 5:  Omega 6:\n')
    fprintf(' (mm)   (rad/s)   (rad/s)   (rad/s)   (rad/s)   (rad/s) \n')
    for x = 1:25:751
        fprintf('%5.1f    %.3f     %.3f     %.3f    %.3f    %.3f \n',R22all(x),omega_2all↙
(x),omega_3all(x),omega_4all(x),omega_5all(x),omega_6all(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR ALPHA 2,3,4,5,AND 6
    fprintf(' R22     Alpha 2:   Alpha 3:    Alpha 4:    Alpha 5:  Alpha 6:\n')
    fprintf(' (mm)   (rad/s^2)  (rad/s^2)  (rad/s^2)  (rad/s^2)  (rad/s^2) \n')
    for x = 1:25:751
        fprintf('%5.1f    %.3f     %.3f     %.3f    %.3f    %.3f \n',R22all(x),accel_2all↙
```

```
(x),accel_3all(x),accel_4all(x),accel_5all(x),accel_6all(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR POSITION,FIRST-ORDER KINEMATIC COEFFICIENTS,AND SECOND-ORDER↙
KINEMATIC COEFFICIENTS
    fprintf(' R22       Xp:        Yp:         Xp'':       Yp'':        Xp":       Yp":\n')
    fprintf(' (mm)     (mm)     (mm/mm)    (mm/mm)  (mm/mm^2)  (mm/mm^2)  (mm/mm^2)\n')
    for x = 1:25:751
        fprintf('%5.1f    %.2f    %.2f     %.3f     %.3f    %.3f    %.3f \n',R22all(x),↙
Xp_all(x),Yp_all(x),Xp_primeall(x),Yp_primeall(x),Xp_2primeall(x),Yp_2primeall(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR UNIT TANGENT AND NORMAL IN THE X AND Y DIRECTIONS
    fprintf(' R22     UtX:       UtY:       UnX:       UnY: \n')
    fprintf(' (mm)   (None)     (None)     (None)     (None) \n')
    for x = 1:25:751
        fprintf('%5.1f    %.3f    %.3f     %.3f     %.3f \n',R22all(x),Ut_X_all(x),↙
Ut_Y_all(x),Un_X_all(x),Un_Y_all(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR RADIUS OF CURVATURE AND POSITION OF CENTER OF CURVATURE IN BOTH X↙
AND Y DIRECTIONS
    fprintf(' R22      rho:       Xcc:        Ycc:     \n')
    fprintf(' (mm)     (mm)       (mm)       (mm) \n')
    for x = 1:25:751
        fprintf('%5.1f    %.2f    %.2f     %.2f  \n',R22all(x),rho_c_all(x),Xcc_all(x),↙
Ycc_all(x))
    end
    fprintf('\n')

    %TABULATED DATA FOR VELOCITY AND ACCELERATION OF POINT P IN BOTH THE X AND Y↙
DIRECTIONS
    fprintf(' R22     VpX:      VpY:      ApX:        ApY \n')
    fprintf(' (mm)    (mm/s)    (mm/s)   (mm/s^2)    (mm/s^2)\n')
    for x = 1:25:751
        fprintf('%5.1f    %.3f    %.3f     %.3f  %.3f  \n',R22all(x),Vp_X_all(x),Vp_Y_all↙
(x),Ap_X_all(x),Ap_Y_all(x))
    end

% PLOTTED DATA
    %PLOT THETA VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,theta_3all,R22all,theta_4all,R22all,theta_5all,R22all,theta_6all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Angle (degrees)')
    title('Angle of Links vs. Length of Input Link')
    legend('Theta 3','Theta 4','Theta 5','Theta 6','Location','northwest')

    %PLOT THETA' VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,theta_3_primeall,R22all,theta_4_primeall,R22all,theta_5_primeall,R22all,↙
theta_6_primeall)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Theta'' (rad/mm)')
    title('Theta'' vs. Length of Input Link')
```

```
    legend('Theta 3''','Theta 4''','Theta 5''','Theta 6''','Location','northeast')

    %PLOT THETA" VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,theta_3_2primeall,R22all,theta_4_2primeall,R22all,theta_5_2primeall,↙
R22all,theta_6_2primeall)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Theta" (rad/mm^2)')
    title('Theta" vs. Length of Input Link')
    legend('Theta 3"','Theta 4"','Theta 5"','Theta 6"','Location','northeast')

    %PLOT OMEGA VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,omega_3all,R22all,omega_4all,R22all,omega_5all,R22all,omega_6all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Omega (rad/s)')
    title('Omega vs. Length of Input Link')
    legend('Omega 3','Omega 4','Omega 5','Omega 6','Location','northeast')

    %PLOT ALPHA VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,accel_3all,R22all,accel_4all,R22all,accel_5all,R22all,accel_6all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Alpha (rad/s^2)')
    title('Alpha vs. Length of Input Link')
    legend('Alpha 3','Alpha 4','Alpha 5','Alpha 6','Location','northeast')

    %PLOT ANGULAR VELOCITY AND ACCELERATION VALUES OF WHEEL AGAINST THE INPUT POSITION OF↙
R22
    figure
    subplot(2,1,1)
    plot(R22all,omega_2all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Angular Velocity of Wheel (rad/s)')
    title('Angular Velocity of Wheel vs. Length of Input Link')
    subplot (2,1,2)
    plot(R22all,accel_2all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Angular Acceleration of Wheel (rad/s^2)')
    title('Angular Acceleration of Wheel vs. Length of Input Link')

    %PLOT P POSITION VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,Xp_all,R22all,Yp_all)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Position (mms)')
    title('Position of P vs. Length of Input Link')
    legend('Xp','Yp','Location','northeast')

    %PLOT P FIRST-ORDER COEFFICIENT VALUES AGAINST THE INPUT POSITION OF R22
    figure
    plot(R22all,Xp_primeall,R22all,Yp_primeall)
    xlabel('Length of Input Link - R22 (mms)')
    ylabel('Position'' (mm/mm)')
    title('Position'' vs. Length of Input Link')
    legend('Xp''','Yp''','Location','northeast')

    %PLOT P SECOND-ORDER COEFFICIENT VALUES AGAINST THE INPUT POSITION OF R22
```

```
figure
plot(R22all,Xp_2primeall,R22all,Yp_2primeall)
xlabel('Length of Input Link - R22 (mms)')
ylabel('Position" (mm/mm^2)')
title('Position" vs. Length of Input Link')
legend('Xp"','Yp"','Location','northeast')

%PLOT THE PATH OF POINT P
figure
plot(Xp_all,Yp_all)
xlabel('Xp (mm)')
ylabel('Yp (mm)')
legend('Path of Point P','Location','northeast')
title('Path of Point P')

%PLOT TANGENT VECTOR VALUES AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,Ut_X_all,R22all,Ut_Y_all)
xlabel('Length of Input Link - R22 (mms)')
legend('Utx','Uty','Location','northeast')
title('Unit Tangent Vectors of Point P')

%PLOT NORMAL VECTOR VALUES AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,Un_X_all,R22all,Un_Y_all)
xlabel('Length of Input Link - R22 (mms)')
legend('Unx','Uny','Location','northeast')
title('Unit Normal Vectors of Point P')

%PLOT THE RADIUS OF CURVATURE AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,rho_c_all)
xlabel('Length of Input Link - R22 (mms)')
ylabel('Radius of Curvature (mms)')
title('Radius of Curvature dependance on Length of Input Link')

%PLOT CENTER OF CURVATURE PATH
figure
plot(Xcc_all,Ycc_all)
xlabel('Center of Curvature X dimension (mms)')
ylabel('Center of Curvature Y dimension (mms)')
title('Center of Curvature')

%PLOT CENTER OF CURVATURE PATH AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,Xcc_all,R22all,Ycc_all)
xlabel('Length of Input Link - R22 (mms)')
ylabel('Center of Curvature (mms)')
title('Center of Curvature dependance on Length of Input Link')

%PLOT VELOCITY OF POINT P AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,Vp_X_all,R22all,Vp_Y_all)
xlabel('Length of Input Link - R22 (mms)')
ylabel('Velocity (mm/s)')
legend('Vpx','Vpy','Location','northeast')
title('Velocity of Point P')
```

```
%PLOT ACCELERATION OF POINT P AGAINST THE INPUT POSITION OF R22
figure
plot(R22all,Ap_X_all,R22all,Ap_Y_all)
xlabel('Length of Input Link - R22 (mms)')
ylabel('Acceleration (mm/s^2)')
legend('Apx','Apy','Location','northeast')
title('Acceleration of Point P')
```